FUNDAMENTALS OF ELECTRICAL CIRCUITS AND SYSTEMS

Energy Science and Technology

Enseignant : **Prof. F. Rachidi**

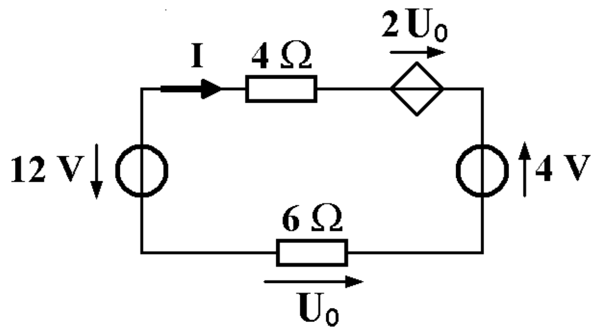
Problem Sets

LAUSANNE, SEPTEMBER 2014

PROBLEM SET I

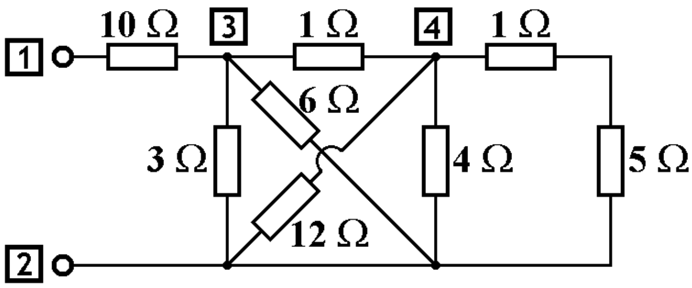
EXERCISE I.1 : KIRCHHOFF'S LAW (1)

Compute U_0 and I in the circuit of the figure below.



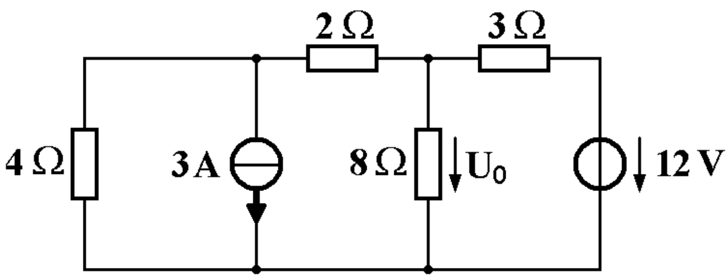
EXERCISE I.2 : EQUIVALENT RESISTANCE

Calculate the equivalent resistance R_{1-2} between points $\boxed{1}$ and $\boxed{2}$ of the figure below.



EXERCISE I.3 : SOURCE TRANSFORMATION

Apply source transformation to compute U_0 in the circuit of the figure below.

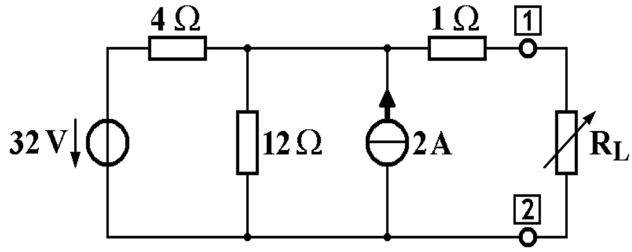


PROBLEM SET II

EXERCISE II.1 : THÉVENIN'S EQUIVALENT CIRCUIT

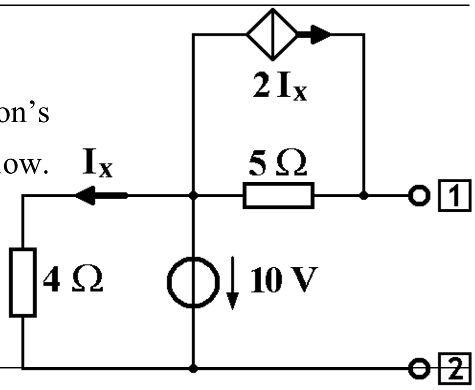
a. Find Thévenin's equivalent circuit, to the left of the terminals $\boxed{1}$ – $\boxed{2}$, in the circuit of the figure below.

b. Calculate the current that is established through a load R_L of 6 Ω, 16 Ω et 36 Ω.



EXERCISE II.2 : NORTON'S THEOREM

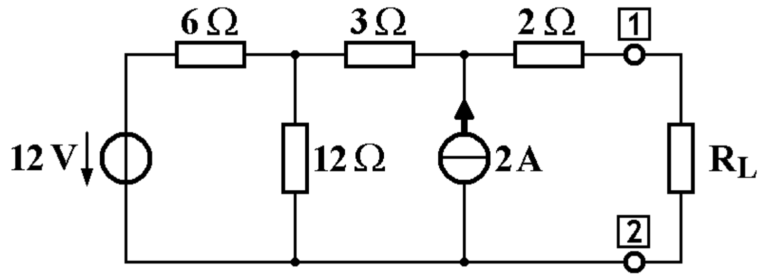
Use Norton's theorem to find Norton's resistance R_N and Norton's current I_N at terminals $\boxed{1}$ – $\boxed{2}$ in the circuit of the figure below.



EXERCISE II.3 : POWER TRANSFER

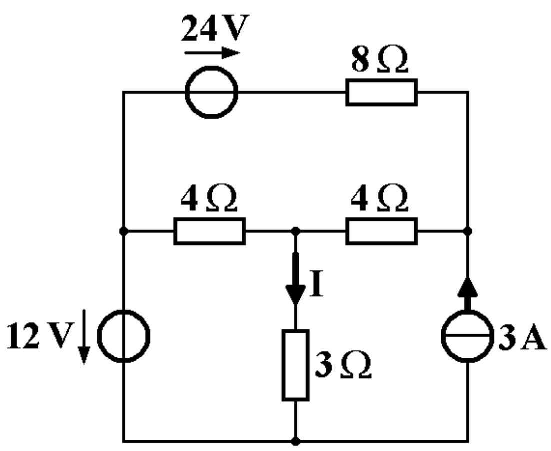
a. Find the value of R_L corresponding to a maximum power transfer in the circuit of the figure.

b. Find the maximum power transferred.



EXERCISE II.4 : SUPERPOSITION THEOREM

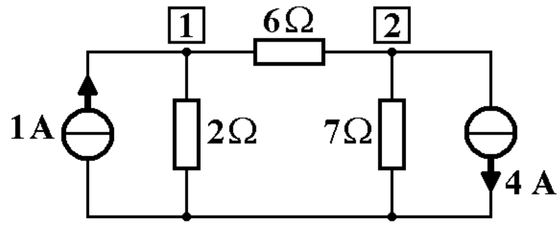
Use the superposition theorem to find I in the circuit of the figure below.



PROBLEM SET III

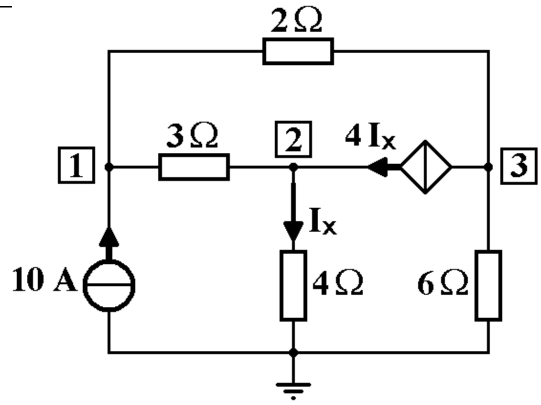
EXERCISE III.1 : NODES VOLTAGES

Calculate the voltages at nodes $\boxed{1}$ and $\boxed{2}$ in the circuit of the figure below.



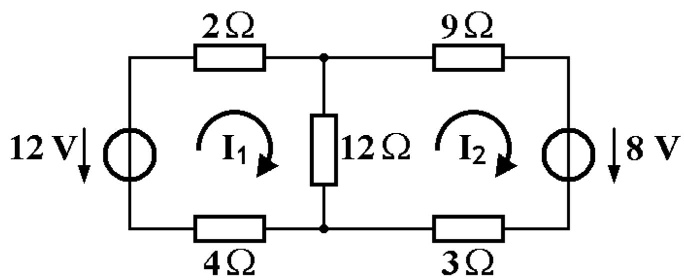
EXERCISE III.2 : NODES VOLTAGES

Calculate the voltages at nodes $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$ in the circuit of the figure below.



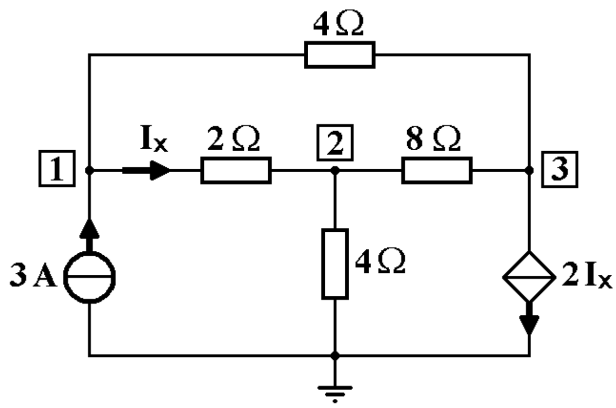
EXERCISE III.3 : MESH CURRENTS

Calculate mesh currents I_1 and I_2 in the circuit of the figure below.



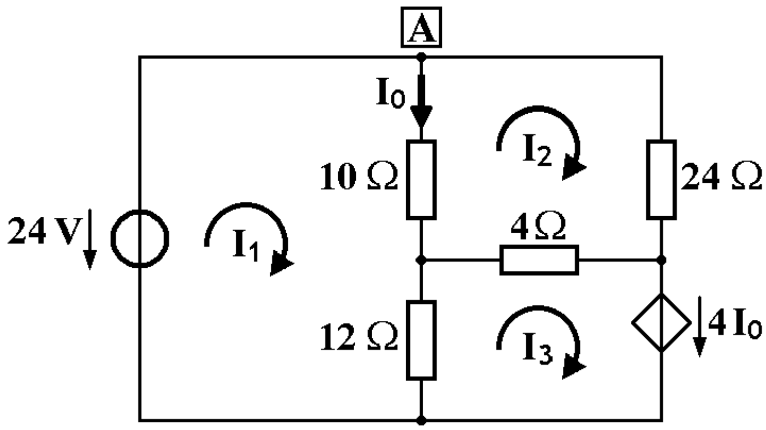
EXERCISE III.4 : NODES VOLTAGES

Calculate the voltages at nodes $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$ in the circuit of the figure below.



EXERCISE III.5 : MESH ANALYSIS

Calculate the current I_o by mesh analysis of the circuit in the figure below.



PROBLEM SET IV

EXERCISE IV.1 : PERIODIC FUNCTIONS

For each of the periodic functions y(t) represented below, compute :

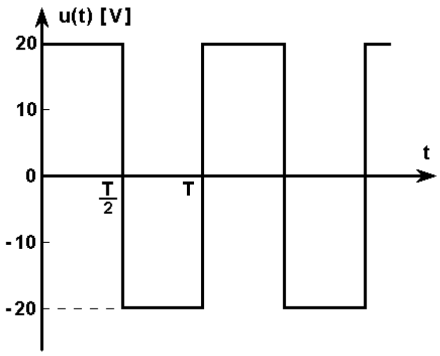
A. the average value : F_m

B. the rms value : F

C. the maximum value (peak value) : F_{max}

D. the minimum value : F_{min}

1



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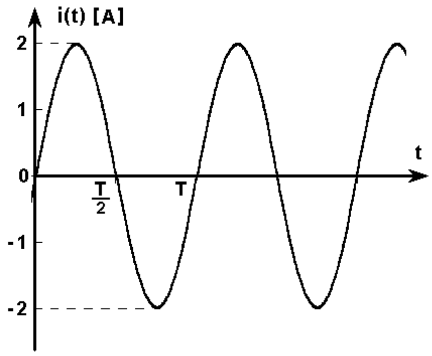

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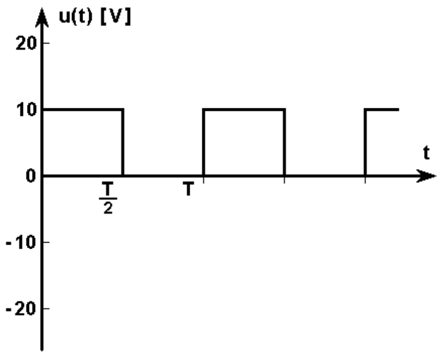

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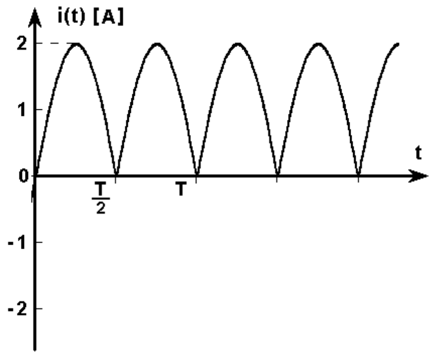

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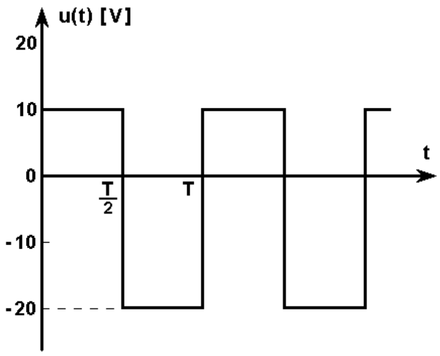

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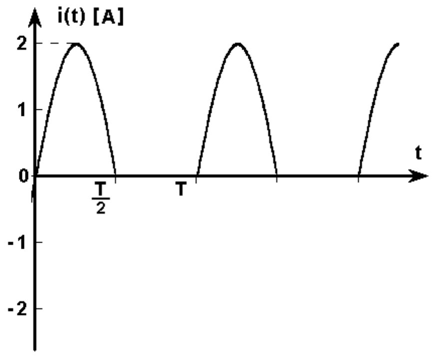
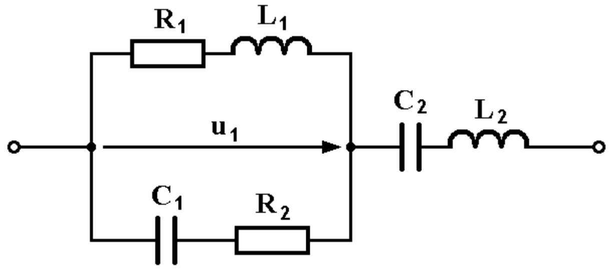


EXERCISE IV.2 : EQUATIONS IN A AC CIRCUIT

The circuit shown below is formed of linear passive elements (resistors, inductors and capacitors).



A. Define all voltages and currents.

B. Write, in instantaneous values, all the equations of the voltages and the currents as well as those which link the voltage at the terminals of each element of the circuit with the current which crosses it.

C. Write the same equations as under 2 using phasors (complex rms values).

D. What does it take to solve the problem numerically in a complex sinusoidal system ?

E. Give the numerical values of all the phasors

Numerical application : $R_1 = 4\ \Omega$ $L_1 = 25$ mH $C_1 = 500\ \mu$F

$R_2 = 8\ \Omega$ $L_2 = 50$ mH $C_2 = 1000\ \mu$F

$u_1 = 20\sqrt{2}\cos(\omega t)$ $\omega = 2\pi f$ and $f = 50$ Hz

F. Draw graphs of the phasors of currents and voltages on a complex diagram.

Scales : U: 1 cm $\leftrightarrow$ 5 V ; I : 1 cm $\leftrightarrow$ 0,5 A.

EXERCISE IV.3 : CALCULATIONS OF IMPEDANCES

At the terminals of each circuit drawn below, a cosine voltage is imposed :

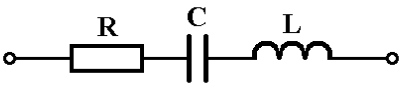
$$u(t) = 4\sqrt{2}\cos(\omega t) \ [V]$$

A. For all circuits, calculate the impedance $\underline{Z}$ (literally and numerically) in the form A + j B and in the form $Z \cdot e^{j\alpha}$.

B. Draw the complex diagram of currents and voltages for each of the circuits.

Numerical application : R = 4 Ω L = 25,5 mH C = 640 µF f = 50 Hz

1



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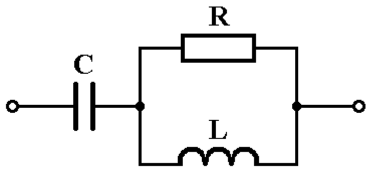

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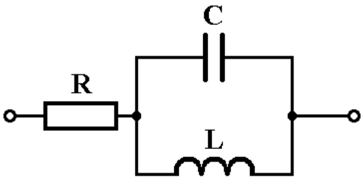

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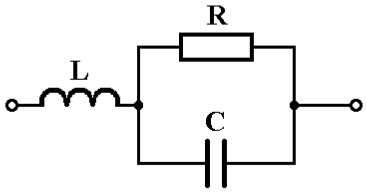

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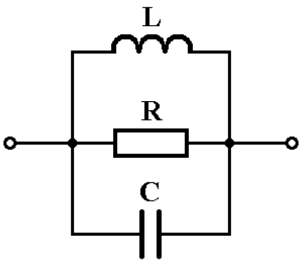

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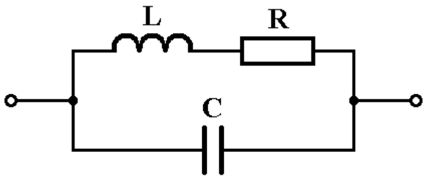

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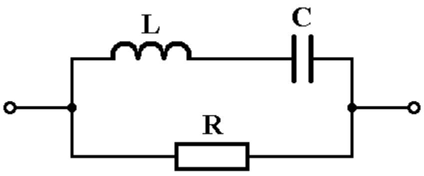
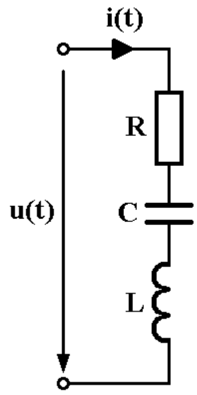


PROBLEM SET V

EXERCISE V.1 : RLC CIRCUIT IN SINUSOIDAL REGIME



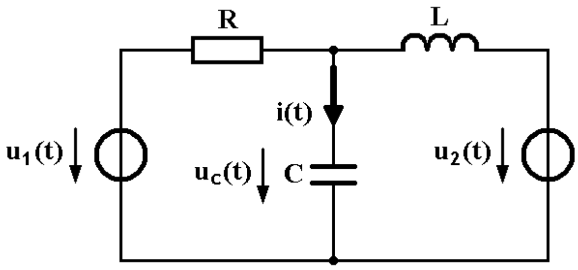
For the electrical circuit below, determine the rms value I of the current i(t), the power factor cosϕ, and the nature of the impedance, for a sinusoidal voltage u(t) of rms value U, for frequencies f_1 and f_2.

Numerical application : $R = 10\ \Omega$ $L = 1\ mH$ $C = 10\ \mu F$

$U = 100\ V$ $f_1 = 1\ kHz$ $f_2 = 2\ kHz$

EXERCISE V.2 : SUPERPOSITION IN SINUSOIDAL REGIME

The following electrical circuit is given, containing two voltage sources of frequencies f_1 and f_2 respectively :



Determine $u_c(t)$ and $i_c(t)$

Numerical application : $R = 30\ \Omega$ $L = 16\ mH$ $C = 300\ \mu F$

$u_1(t) = 100\sqrt{2}\sin\omega_1 t$ $u_2(t) = 100\sqrt{2}\sin\omega_2 t$

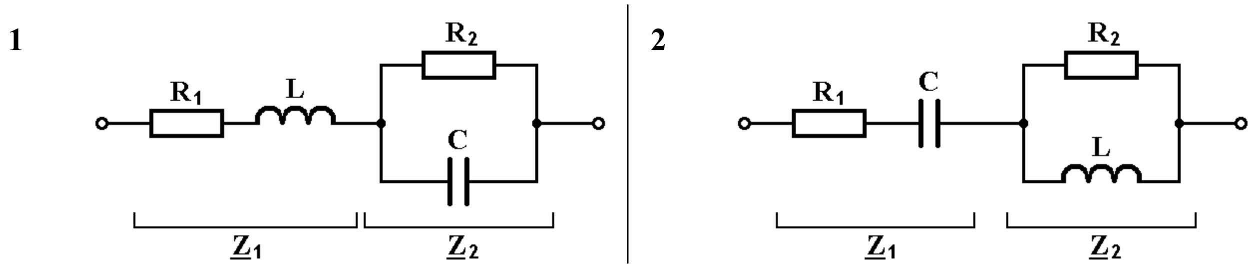
$\omega_k = 2\pi f_k$ $f_1 = 50\ Hz$ $f_2 = 100\ Hz$

In trigonometric calculations, calculators are not able to distinguish angles α and α + 180°.

PROBLEM SET VI

EXERCISE VI.1 : IMPEDANCES LOCUS

Let the two circuits represented below :



A. Represent, in the complex plane, the admittance locus of $\underline{Y}_2 = 1/\underline{Z}_2$ and of impedances $\underline{Z}_1$, $\underline{Z}_2$ and $\underline{Z}_{total}$, for frequency f ranging from 0 to ∞.

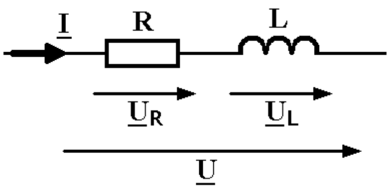
B. Indicate for which values of f the circuit is inductive, resistive or capacitive.

Numerical application : $R_1 = 10\ \Omega$ $R_2 = 33\ \Omega$ $L = 6{,}5$ mH $C = 12\ \mu$F

Scales : Z : 1 cm ↔ 5 V/A Y : 1 cm ↔ 0,01 A/V

EXERCISE VI.2 : POWER CALCULATION

The circuit shown below is subjected to a sinusoidal voltage :



A. Calculate the total impedance in literal form.

B. Calculate the numerical complex value (polar form and cartesian form) of the total impedance.

C. Calculate the inductance L.

D. Calculate the rms value of $\underline{U}$.

E. Represent the complex diagram of current and voltages.

F. Calculate the active and reactive powers consumed by the circuit.

G. Calculate the value of the capacitor C to be connected in parallel with the RL branch, to cancel the reactive power (compensation of the reactive power.)

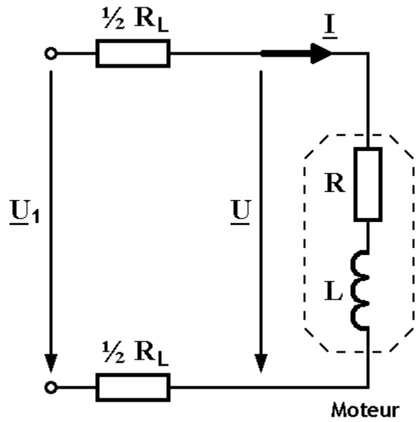
Numerical application : $U_R = 60$ V $U_L = 80$ V $I = 2$ A (rms values)

f = 50 Hz

PROBLEM SET VII

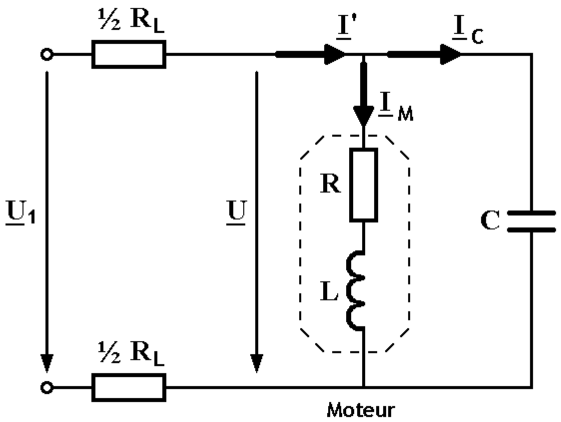
EXERCISE VII.1 : INCREASE OF COSϕ OF A RL LOAD

The electric motor shown here works at an active power P and with the power factor $\cos\phi$. The voltage at the motor terminals is U.



A. Calculate the value of the R and L elements of the motor equivalent diagram, as well as the total current $\underline{I}$ flowing in the line.

To increase the power factor $\cos\phi = 0,6$ to $\cos\phi = 0,95$ at the motor, a group of capacitors, total capacity C, is connected in parallel with the motor, as shown below.



B. Calculate the value of the capacity, the voltage U remaining the same.

C. Calculate the resulting total current I'.

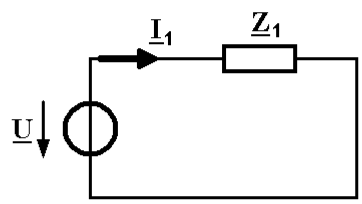
D. Calculate the active power losses in the lines, P_{pertes} et P'_{pertes} in both cases (with and without capacitor).

E. What is the effect of an increase of the power factor $\cos\phi$?

Numerical application : $P = 45{,}5$ kW $U = 380$ V $R_L = 0{,}05\ \Omega$
$f = 50$ Hz $\cos\phi = 0{,}6$

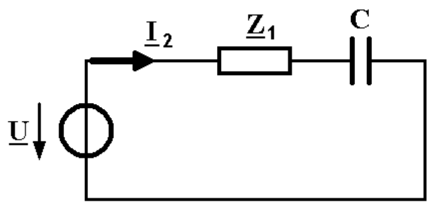
EXERCISE VII.2 : POWER IN ALTERNATIVE REGIME

Consider an impedance $\underline{Z}_1$, powered by a sinusoidal voltage of rms value U = 100 V; frequency = 50 Hz.



In this case, we measure a current $I_1 = 10$ A and $\cos\phi = 0{,}8$.

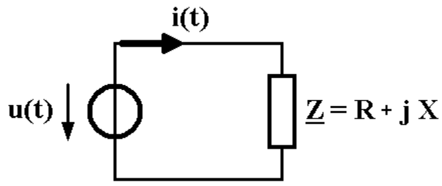
In a second step, a capacitor C is added in series with the impedance $\underline{Z}_1$ and then we measure a $\cos\phi = 0{,}6$.



Calculate R_1, X_1, C and I_2 (all solutions)

EXERCISE VII.3 : ACTIVE AND REACTIVE POWER

A single-phase AC source supplies an unknown impedance : $\underline{Z} = R + jX$.



We measure : $u(t) = 20 \cdot \sin(5000 \cdot t - \pi/3)$

$i(t) = 12 \cdot \sin(5000 \cdot t - \pi/18)$

A. Is the impedance capacitive or inductive ?

B. Calculate the apparent, active and reactive powers absorbed or produced by the impedance.

C. Calculate R and X.

PROBLEM SET VIII

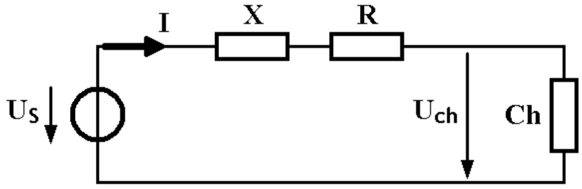
EXERCISE VIII.1 : ACTIVE AND REACTIVE POWER

In the electrical diagram below, a single-phase AC source supplies a load (Ch) via a reactance (X) and a resistor (R). Upstream of the circuit, an active power output is measured P_s and a reactive power Q_s.

$U_s = 12{,}47$ kV

$X = 15\ \Omega$ $R = 2{,}4\ \Omega$

$P_s = 3$ MW $Q_s = 2$ Mvar



A. Calculate the phase difference between the current and the supply voltage.

B. Calculate U_{ch}, P_{ch} and Q_{ch} (voltage, active power and reactive power on the load).

C. Calculate the phase difference between U_s and U_{ch}.

EXERCISE VIII.2 : ACTIVE AND REACTIVE POWER

An impedance is given by its module and phase shift : $|\underline{Z}| = 33\ \Omega$ and $\varphi = 30°$.

A. Calculate its resistance and reactance.

B. Calculate the active, reactive and apparent powers for an applied rms voltage U = 230 V

EXERCISE VIII.3 : APPARENT POWER

The current and active power of two systems are measured when powered by a single-phase sinusoidal source U = 230 V :

<u>System 1</u> : $I_1 = 8$ A ; $P_1 = 1{,}3$ kW System 2 : $I_2 = 10$ A ; $P_2 = 1{,}7$ kW

A. Calculate reactive and apparent powers for both systems: Q_1, Q_2, S_1, S_2.

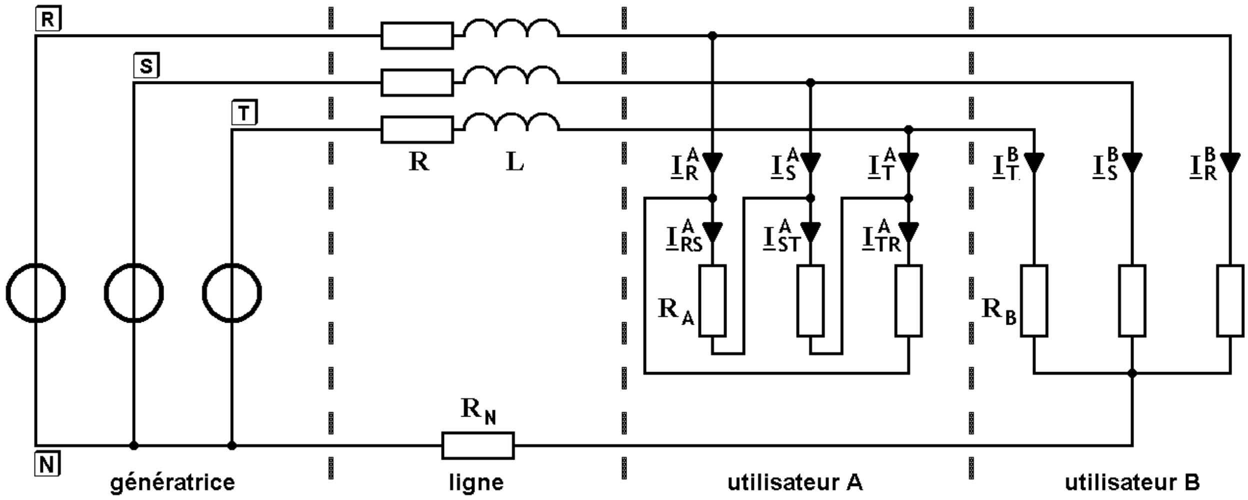
B. Calculate the apparent power when the two systems are connected in parallel, the whole being powered by 230 V.

C. Same question when both systems are connected in series, the whole being powered by 230 V.

PROBLEM SET IX

EXERCISE IX.1 : BALANCED THREE-PHASE SYSTEM

Let the three-phase system below :



An ideal three-phase generator requires a symmetrical three-phase system of phase voltages of rms value U, at a frequency f. It feeds two symmetrical three-phase users through a line of length d, each phase conductor having a per-unit-length resistance R' and a per-unit-length inductance L'.

User A has three resistors R_A built in triangle and user B, three resistors R_B built in star. The return conductor has a resistance R_N.

A. Calculate the current supplied by each phase of the generator (module and phase shift with respect to the phase voltage of the same phase).

B. Calculate the active and reactive powers provided by the generator.

C. Calculate the active and reactive powers consumed by consumer A and consumer B.

Numerical application : U = 236 V f = 50 Hz
d = 5 km R' = 0,04 Ω/km L'= 0,8 mH/km
R_A = 10,8 Ω R_B = 3,6 Ω R_N = 2 Ω

EXERCISE IX.2 : COMPENSATION OF THE REACTIVE POWER IN THREE-PHASE SYSTEMS

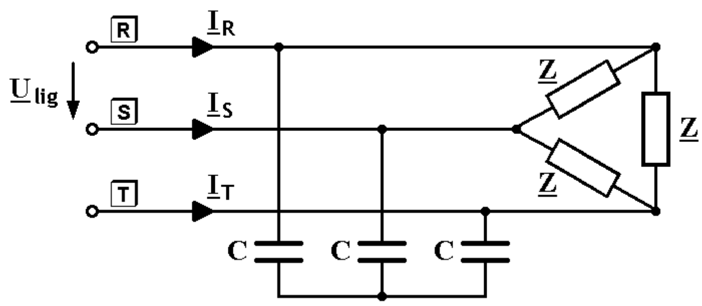
A three-phase user has three inductors equal to $\underline{Z}$, built in triangle, and which each presents a module of 10 Ω and an inductive power factor cosϕ = 0,8. Line voltages have a rms value U_{lig} = 400 V and a frequency f = 50 Hz.

A wye-shaped capacitor bank C is installed so that the apparent cosϕ = 1.

A. Calculate C.

B. Calculate the active power supplied by the network.

C. Calculate line currents, <u>without</u> capacitors and <u>with</u> capacitors.



PROBLEM SET X

WYE-DELTA TRANSPOSITION

There are three common problems, which involve transposing a delta connection to a wye connection or vice-versa :

- The problem of constant impedance power change.
- The problem of adaptation to two different networks.
- The problem of equivalent impedances at constant powers.

In all three cases, it is a balanced three-phase system. **So just do the calculations for a single phase, then generalize the result.**

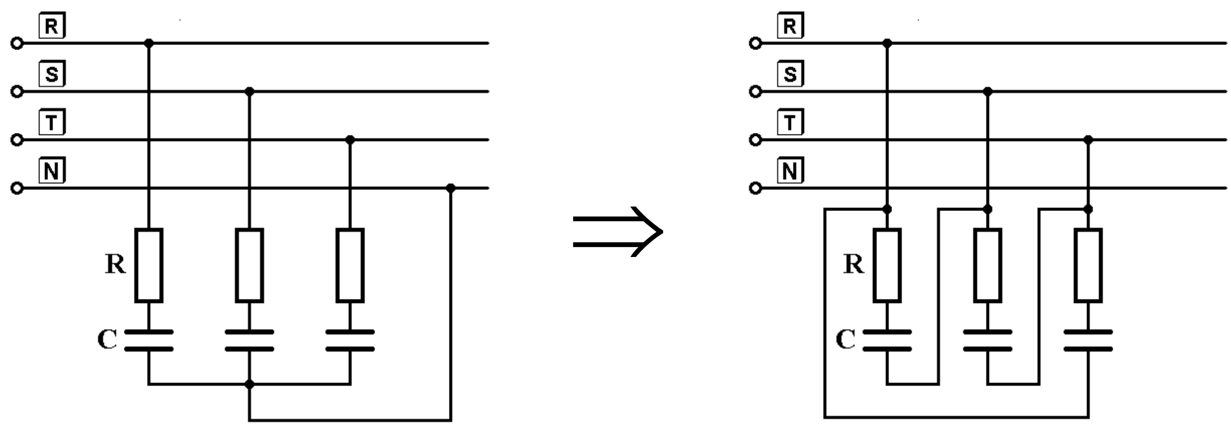
☞ When a single voltage is indicated, it is by convention the rms value of the line voltage.

EXERCISE X.1 : CHANGE OF POWER

In this type of problem, the network and the load impedances of the user remain the same. The user shown below is first connected in star to a 400 V / 50 Hz network.

A. What happens if we remove the connection to the neutral conductor ?

We then connect the same user (same R and C loads) in triangle on the same network.



In each case :

B. What is the load voltage ?

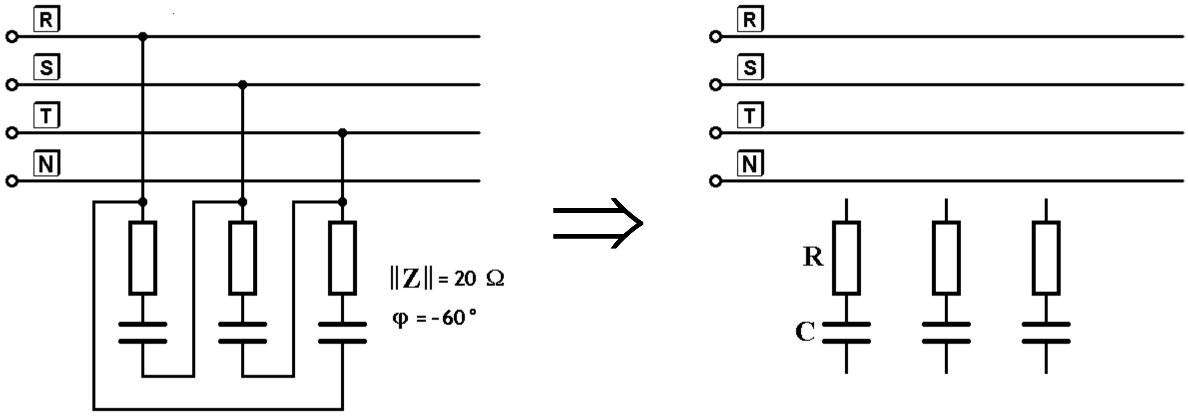
C. Calculate the current flowing in each of the phases (modulus and phase shift with respect to the phase voltage).

D. Calculate the active and reactive powers absorbed by a phase and by all the phases.

Numerical application : $R = 10\ \Omega$ $C = 185\ \mu F$

EXERCISE X.2 : ADAPTATION TO TWO DIFFERENT NETWORKS

The user shown below was initially connected in triangle on a 230 V / 50 Hz network. When the network went to 400 V / 50 Hz, the user was connected in star.



A. Complete the diagram.

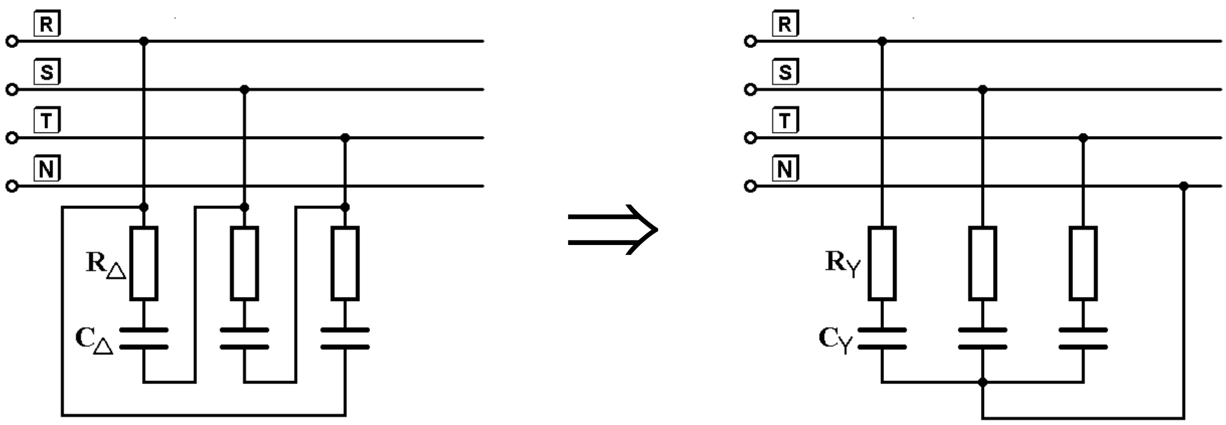
B. Check that the consumer draws the same active and reactive powers in both cases.

C. Calculate the current in the line, for both cases.

EXERCISE X.3 : EQUIVALENT IMPEDANCES

When solving a problem of electrical engineering, it may be advantageous to replace a three-phase load in triangle with an equivalent three-phase load in star.

☞ The charges are said to be equivalent if, from the outside, they behave in the same way: for the same phase voltage, they consume the same line current and the same active power.



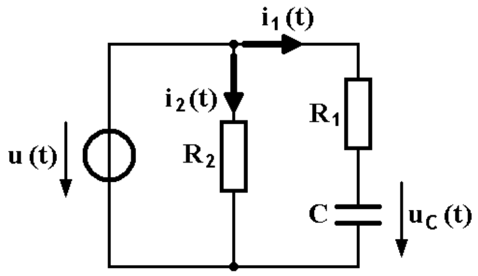
A. Calculate the elements of the charge in star, so that the powers consumed remain the same as for the connection in triangle.

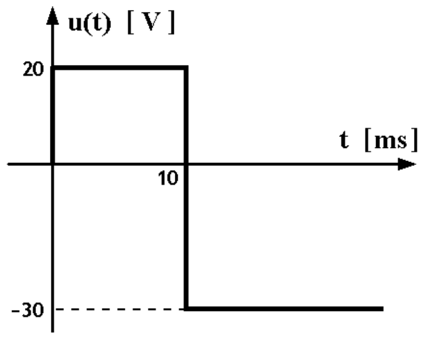
Numerical application : $R_{\Delta} = 21\ \Omega$ $C_{\Delta} = 180\ \mu F$

PROBLEM SET XI

EXERCISE XI.1 : RC CIRCUIT IN TRANSIENT REGIME (1)

Let the RC circuit shown below, with the voltage source u(t).





A. Calculate the current $i_1(t)$.

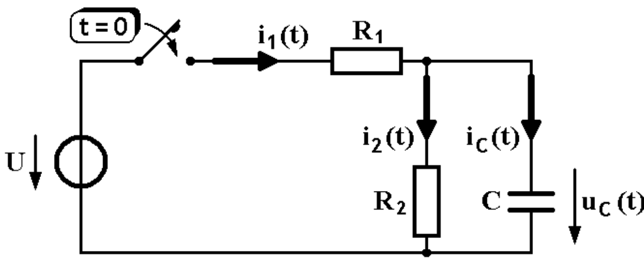
B. Calculate the voltage $u_C(t)$.

C. Draw $u_C(t)$ and $i_1(t)$, from 0 to 20 ms, clearly indicating limit values and time constants.

Numerical application : $R_1 = 500\ \Omega$ $R_2 = 100\ \Omega$ $C = 5\ \mu F$ $u_C(0) = 5\ V$

EXERCISE XI.2 : RC CIRCUIT IN TRANSIENT REGIME (2)

The RC circuit shown below is powered by a constant voltage U. At time t = 0, the switch is closed and then opened again at $t_1 = 1$ ms.



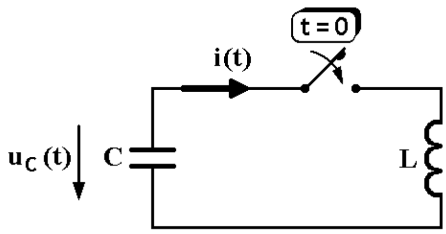
A. Calculate and represent the voltage $u_C(t)$ as a function of time, from 0 to 2 ms.

B. What are the time constants of the circuit ?

Numerical application : $R_1 = 10\ \Omega$ $R_2 = 15\ \Omega$ $C = 10\ \mu F$
$U = 100\ V$ $u_C(0) = 0\ V$

EXERCISE XI.3 : LC CIRCUIT IN TRANSIENT REGIME

Let the LC circuit represented below.

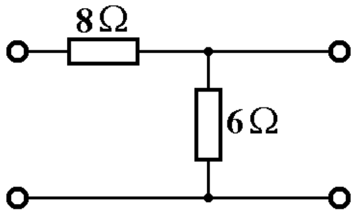


A. Calculate the current i(t) for t > 0, knowing that $u_C(t) = U_o$ for t < 0

PROBLEM SET XII

EXERCISE XII.1 : QUADRIPOLE

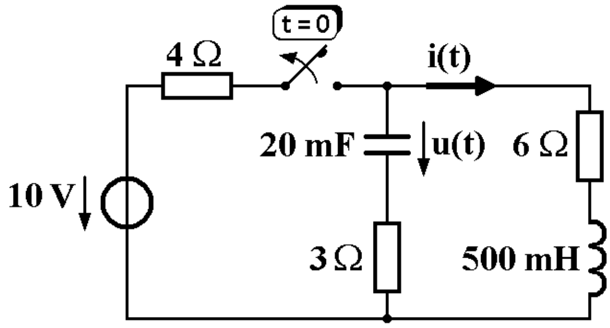
Find the impedance parameters of the quadripole in the figure opposite.



EXERCISE XII.2 : OPENING A SWITCH

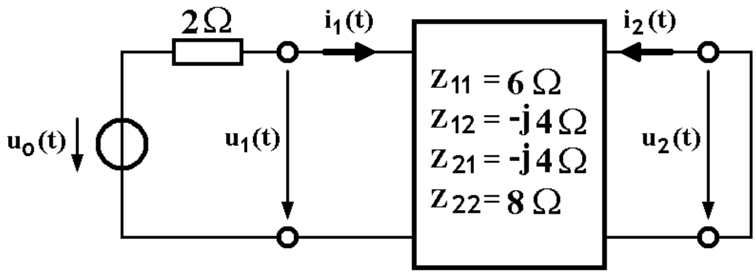
The circuit of the figure opposite is in a steady state immediately before the opening of the switch, at time t = 0.

Calculate the current i(t) for t ≥ 0



EXERCISE XII.3 : QUADRIPOLE

Calculate the currents I_1 and I_2 in the quadripole of the figure opposite.



EXERCISE XII.4 : T-DIAGRAM

Calculate the admittance parameters and the transmission parameters for the T-diagram of the figure opposite.